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matically straight. This can be done by a linkwork so constructed that when the whole system is moved there will be three points, as O, P, and Q collinear in every position, and in the relation $OP\times OQ=$constant.

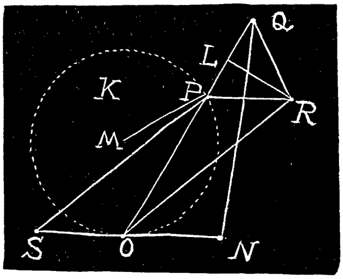
A linkwork that will describe the inverse of a given curve or surface is termed an Inversor.

The inverse points as O, P, and Q, in an inversion are called the foci.

THEOREM. *Let the side OS of the parallelogram ORPS be extended to N, making ON equal to OS, and located so that NQ equals OR, and RQ equals PR. Then if this figure be embodied in a linkage:*

1. *The points O, P, and Q, are always collinear.*
2. *The points O, P, and Q, are inverse points.*
3. *If the focus O of the linkage be pivoted to a fixed base, and the focus P be constrained to move in a circle through O, the focus Q will describe a straight line.*

Given: $SP=OR=NQ$; $SO=ON=PR=RQ$.



Proof 1. $\triangle ONR \cong \triangle QNR$.

But $OP \parallel NR$.

$\therefore$ The points O, P, and Q are collinear.

Proof 2. Draw RL perpendicular to OPQ. Then $OP=OL-LP=\sqrt{(OR^2-RL^2)}-\sqrt{(PR^2-RL^2)}$; $OQ=OL+LP=\sqrt{(OR^2-RL^2)}+\sqrt{(PR^2-RL^2)}$.

$\therefore OP.OQ=OR^2-PR^2=$a constant.

$\therefore$ The points O, P, and Q are inverse points.

Proof 3. $\because$ O, P, and Q are inverse points, and since a straight line is the inverse of a circle through the pole of inversion,

$\therefore$ The line QV is a straight line, the inverse of the circle K. Q. E. D.

SIX PROPOSITIONS ON PRIME NUMBERS.

By R. D. CARMICHAEL, Hartselle, Ala.

PROPOSITION A. *If p is an odd prime,* $(1.2.3\ldots\frac{p-1}{2})^2+(-1)^{\frac{1}{2}(p-1)}\equiv 0$ *(mod p)*.

By Wilson's theorem, if p is prime

$$1.2.3\ldots(p-1)+1\equiv 0 \pmod p \qquad (1).$$

This may be written

$$(1.2.3\ldots\frac{p-3}{2}.\frac{p-1}{2})(p-\frac{p-1}{2})(p-\frac{p-3}{2})\ldots(p-2)(p-1)+1\equiv 0 \pmod p \qquad (2).$$

Expanding and casting out all terms containing p as a factor (p being an odd prime),

$$(1.2.3.....\frac{p-1}{2})^2+(-1)^{\frac{1}{2}(p-1)}\equiv 0 \pmod p \qquad (3).$$

PROPOSITION B. *If* $(1.2.3.....\frac{p-1}{2})^2+(-1)^{\frac{1}{2}(p-1)}\equiv 0$ *(mod* p*),* p *is prime.*

It is evident that we may revert from (3) to (2) by re-inserting the cast out terms containing p as a factor. (2) is but another form of (1). Hence, we have only to verify the well-known theorem that p is a prime, if

$$1.2.3.....(p-1)+1\equiv 0 \pmod p \qquad (4)$$

For any value of p *not* prime, except $p=4$, it is easily shown that

$$1.2.3.....(p-1)\equiv 0 \pmod p \qquad (5).$$

Hence, when (4) is true, p is prime.

PROPOSITION C. *If* $4n+1$ *is prime, it may be expressed as the sum of two parts* r *and* s *such that* $r^2+1\equiv 0$, $s^2+1\equiv 0$, *and* $rs-1\equiv 0$ *(mod* p*).*

For p a prime of the form $4n+1$, (3) may be written

$$(1.2.3.....2n)^2+1\equiv 0 \pmod{4n+1} \qquad (6).$$

This may be written in either of the forms,

$$[l(4n+1)+r]^2+1\equiv 0 \pmod{4n+1} \qquad (7),$$
$$[(l+1)(4n+1)-s]^2+1\equiv 0 \pmod{4n+1} \qquad (8),$$

where l is an integer and $r+s=4n+1$. Hence, expanding and casting out the terms containing $4n+1$ as a factor, we get

$$r^2+1\equiv 0,\ s^2+1\equiv 0 \pmod{4n+1} \qquad (9),$$

from which easily follsws $rs-1\equiv 0$.

PROPOSITION D. *If* $a+1$ *and* $2a+1$ *are both primes,* $(1.2.3.....a)^4-1\equiv 0$ $[$*mod* $(a+1)(2a+1)]$.

By Wilson's theorem,

$$1.2.3....\,a+1\equiv 0 \pmod{a+1} \qquad (10).$$
$$\therefore (1.2.3.....a)^4-1\equiv 0 \pmod{a+1} \qquad (11).$$

By Proposition A above,

$$(1.2.3.....a)^2+(-1)^a\equiv0 \pmod{2a+1} \qquad (12).$$

$$\therefore (1.2.3.....a)^4-1\equiv0 \pmod{2a+1} \qquad (13).$$

Since $a+1$ and $2a+1$ are prime to each other, (11) and (13) lead to the theorem stated.

PROPOSITION E. *If* $(1.2.3.....a)^4\equiv0$ $[mod\ (a+1)(2a+1)]$, $a+1$ and $2a+1$ *are both prime.*

We have to show that $a+1$ and $2a+1$ are prime when (11) and (13) hold.

If $a+1=4$, (11) does not hold. For all other values of $a+1$ *not* prime, $1.2.3.....a\equiv0 \pmod{a+1}$. Hence, when $a+1$ is not prime, (11) does not hold. Therefore, $a+1$ is prime under the given condition.

If $2a+1$ is *not* prime, $1.2.3.....2a\equiv0 \pmod{2a+1}$.

$$\therefore (1.2.3.....(a-1)a.[(2a+1)-a][(2a+1)-(a+1)].....$$
$$.....[(2a+1)-2][(2a+1)-1]\equiv0 \pmod{2a+1}.$$

Expanding and casting out terms containing $2a+1$ as a factor, we get

$$(1.2.3.....a)^2\equiv0 \pmod{2a+1} \qquad (14),$$

if $2a+1$ is *not* prime. Hence, when (13) holds, as (14) does not then hold, $2a+1$ is odd.

PROPOSITION F. *If* $a+1$ *and* $2a+1$ *are both prime,* $(a+1)(2a+1)$ *may be expressed as the sum of* r *and* s *such that* $r^4-1\equiv0$, $s^4-1\equiv0$, *and* $r^2s^2-1\equiv0$ $[mod\ (a+1)(2a+1)]$.

The demonstration depends upon Proposition D above, and is similar in method to that of Proposition C. It may easily be supplied by the reader.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

No. 223 was also solved by L. E. Newcomb.

Mr. L. S. Shively calls attention to the fact that a solution of No. 225 is given in C. Smith's "A Treatise on Algebra," page 188, Ex. 4.

226. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Find the real roots of the system

$$x^2+w^2+v^2=a^2,\quad vw+u(y+z)=bc,$$
$$w^2+y^2+u^2=b^2,\quad wu+v(z+x)=ca,$$
$$v^2+u^2+z^2=c^2,\quad uv+w(x+y)=ab.$$